General Physics Notes (07102026):

We are investigating toroidal coordinates. Let's think about the rotating torus in spaceflight used for artificial gravity, the tokamak, and inductor coils in toroidal shapes.

$$x = \left[\frac{a \sinh \nu \cos \phi}{\cosh \nu - \cos u}\right] \tag{1}$$

$$y = \left[\frac{a \sinh \nu \sin \phi}{\cosh \nu - \cos u}\right] \tag{2}$$

$$z = \left[\frac{a \sin u}{\cosh \nu - \cos u}\right] \tag{3}$$

These relations give us a description of the toroidal geometry we need. Hopefully, they'll be useful with geodesics, MHD optimization, coil design, and MD simulations.

An important relation is the Laplacian. Quick and dirty. Here for the Poisson equation in toroidal coordinates we have the expression.

$$\nabla^2 \Phi = \frac{\operatorname{csch} \nu (\cosh \nu - \cos u)^3}{a^2} \cdots$$
$$\left[\frac{\partial}{\partial u}\frac{\sinh \nu}{\cosh \nu - \cos u}\frac{\partial}{\partial u} + \frac{\partial}{\partial \nu}\frac{\sinh \nu}{\cosh \nu - \cos u}\frac{\partial}{\partial \nu} + \frac{\partial}{\partial \phi}\frac{\operatorname{csch} \nu}{\cosh \nu - \cos u}\frac{\partial}{\partial \phi}\right]\Phi$$

For the record, this is equation (4) for this document.