General Physics Notes (07072026):

The two most important problems to solve in the magnetic thermonuclear fusion reactor problem are ignition and turbulence. Calculate the plasma frequency with the ion motion included, thus justifying our assumption that the ions are fixed. Generally, we have a few relations that help us define plasma oscillations in a linearized plasma.

$$\omega_p = \left(\frac{n_x e^2}{\epsilon_0 m}\right)^{1/2} \tag{1}$$

We also use classical electrodynamics.

$$\nabla^2 \phi = \frac{\rho}{\epsilon_0} \tag{2}$$

$$\vec{F} = \frac{\partial \vec{p}}{\partial t} = q\vec{E} + q(\vec{v} \times \vec{B}) \tag{3}$$

There is also continuity.

$$\nabla \rho + \frac{\partial v}{\partial t} \rho = 0 \tag{4}$$

These three equations solve almost all of our problems and neglecting the new zeta calculus and the emergence of scale invariant turbulence, we have a linearized plasma wave setup or ensemble that neglects quantum effects at very low temperatures, energies, and pressures. Thus, we use statistical thermodynamics and introduce $\hbar$ much later. We assume $1-D$ motion.

$$\epsilon_0 \nabla^2 \phi = \epsilon_0 \vec{\nabla} \vec{E} = \epsilon_0 \frac{\partial}{\partial x} E = q(n_i - n_e) \tag{5}$$

This difference between the n's is the number density which again can conform to a zeta turbulent prime model. This is something we should discuss.

$$(n_i - n_e) = \sum_{j=0}^{\infty} n_j \zeta(s', t') \tag{6}$$

This is just a rough and dirty approximation for charged species density and will be developed with the scale invariant quantum foam, turbulent quark gluon plasma, nebular hypothesis, cluster theory, and inflationary model which involves some CMB calculations. We have a lot to unpack in the plasma model and the ζ additions. Perturbations define the system for small oscillations, hence $n_i = n_{i0} + n_{i1}$ and $n_e = n_{e0} + n_{e1}$ or something like that.

$$n_x = n_{x0} + \sum_{j=0}^{\infty} \zeta(s', t') n_{xj} \tag{7}$$

I like to think of infinite order perturbation in the Mlodinow approach although there is not much in the infinite dimensional approach that is relevant or inherently practical in plasma frequency applications. The real trick is always reducing the infinite complexity to the infinite simplicity, the isomorphism of information density in the description. The electron and ion EOM is as follows.

$$m_{e,i}n_{e,i}\left(\frac{\partial v_{e,i}}{\partial t}+(v_{e,i}-\nabla)v_{e,i}\right)=en_{e,i}E(en_{e,i}(\vec{v}_{e,i}\times\vec{B})) \tag{8}$$

We can use the linear expansion and get more relations. (I'll leave this as an exercise to the reader.) From the Lorentz force term and the continuity terms we find ion exchange and motion.

$$n_{e1}=\left(\frac{k}{\omega^2}\right)n_0 i\left(\frac{e}{m_e}\right)E_1 \tag{9}$$

$$n_{i1}=\left(\frac{k}{\omega^2}\right)n_0 i\left(\frac{e}{m_i}\right)E_1 \tag{10}$$

We get a relation.

$$n_{e1}+n_{i1}=\frac{k}{\omega^2}n_0 ieE_1\left(\frac{1}{m_e}+\frac{1}{m_i}\right) \tag{11}$$

Then with some algebra we can find from the plasma frequency another relation that can then be simplified.

$$\frac{n_0 e}{\epsilon_0}\left(\frac{1}{m_e}+\frac{1}{m_i}\right) \tag{12}$$

Hence...

$$\omega_T^2=\omega_p^2+\Omega^2 \tag{13}$$