General Physics Notes (07162026):

Particle models include all collisions where plasma models only include a damping term; scattering must be fundamental in the particle model when it surpasses the plasma model. Let's find the dispersion relation for electrostatic electron waves propagating at an arbitrary angle θ relative to B_0 and show $\omega^2(\omega^2 - \omega_h^2) + \omega_c^2\omega_p^2 = 0$, then write out the two solutions and show when $\theta \to 0$ and $\theta \to \frac{\pi}{2}$ our previous results are recovered. Complete the square to show an ellipse forms when $x = \cos\theta$ and $y = \frac{2\omega^2}{\omega_{h2}}$. Now plot an ellipse for $\frac{\omega_p}{\omega_c} = 1, 2, \ldots \infty$. Here, show if $\omega_c > \omega_p$ the lower root for ω is always less than ω_p for any $\theta > 0$ and upper root always lies between ω_c and ω_h and that if $\omega_p > \omega_c$ the lower root lies below ω_c while the upper root is between ω_p and ω_h.

$$\frac{\partial\omega}{\partial k} = v_g \qquad \frac{\partial E}{\partial k} = f(\omega) \tag{1}$$

We can really expand the structure of energy dispersion from a fractional dimensional wave picture.

$$\frac{\partial E}{\partial k} = \omega r^D \zeta(r,t) e^{ik\cdot r - i\omega t} \tag{2}$$

Here, the ζ contains the prime distribution and turbulence associated with real stochastic dispersion and scattering.

$$\frac{\mathbb{I}f(\omega)}{\mathbb{R}f(\omega)} = \delta(phase) \tag{3}$$

We'll evaluate the Lorentz force relation. Unfortunately is it linearized and we cannot use the zeta turbulent density.

$$m_e(n_0 + n_1)(i\omega)\vec{v}_1 = q_e\vec{E} + q_e(\vec{v}_1 \times \vec{B}) \tag{4}$$

Simplified with the traditional rules of multivariable calculus.

$$im_e n_1 \omega v_1 = q_e E_x + q_e E_y + q_e(v_1 B_0 \sin\theta) \tag{5}$$

We also know the continuity.

$$\frac{\partial\rho}{\partial t} + \nabla v = 0 \tag{6}$$

Again we develop this from linearization. $\rho = n_0 + n_1$. We ignore higher order terms.

$$n_0 m_e \frac{\partial\vec{v}}{\partial t} = q_e\vec{E} + q_e(\vec{v} \times \vec{B}) \tag{7}$$

We also can find the Poisson equation and the linear charge density. More exotic densities might be useful in computational modeling. We'll find something more.

$$-m_e n_0(i\omega)\vec{v}_1 = q_e n_0\vec{E}_1 + q_e n_0(\vec{v}_1 \times \vec{B}_0) \tag{8}$$

The velocities related to the frequency are following.

$$v_x = -\frac{iqE_x}{m_e\omega}\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)^{-1} \qquad v_y = \frac{i_c}{\omega}v_x \quad v_z = -\frac{iq_e}{m_e\omega}E_z \tag{9}$$

$$\vec{\nabla}(n\vec{v}) + \frac{\partial n}{\partial t} = 0 \tag{10}$$

We generate this.

$$n = n_0 + n_1 \quad \rightarrow \quad n = \sum_{i=0}^{\infty} n_i \zeta(s_i, t_i) \tag{11}$$

$$n_1 = -\frac{n_0}{\omega}\left(\frac{iq_e}{\omega m_e}\left(k_x E_x \left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)^{-1} + k_z E_z\right)\right) \tag{12}$$

$$k_x = k\sin\theta \qquad k_z = k\cos\theta \tag{13}$$

After some algebraic manipulation and a little heartache...

$$\omega^2(\omega^2 - \omega_h^2) + \omega_p^2\omega_c^2\cos^2\theta = 0 \tag{14}$$

We can simplify this and it is left as an exercise to do it. If you need it, then ask me and I'll show you the handwritten notes and the graph. Basically, the quadratic equation is recovered and the equation of the ellipse is formed.

$$(\omega^2 - \frac{1}{2}\omega_h^2) + (\omega_p\omega_c\cos\theta)^2 = \frac{1}{2}\omega_h^2 \tag{15}$$

I'll just go over the main equation that gives the upper and lower frequencies and their boundaries.

$$\omega^2 = \frac{1}{2}(\omega_p^2 + \omega_c^2) \pm [(\omega_p^2 + \omega_c^2)^2 - 4\omega_p^2\omega_c^2\cos^2\theta]^{\frac{1}{2}} \tag{16}$$